No. of Printed Pages : 5

Sem-I-Math-CC-I(R&B)

2023

Time - 3 hours

Full Marks - 60

Answer ***all groups*** *as per instructions.*

Figures in the right hand margin indicate marks.

The symbols used have their usual meaning.

GROUP - A

1. Answer all questions and fill in the blanks as required. [1 × 8

(a) Sin ho – cos ho = ________.

(b) $\lim\limits_{x \to \infty} \dfrac{\log x}{x}$ = ________.

(c) $\int x\, e^x\, dx$ = ________.

(d) How many loops are there in the curve $r = a \sin 2\theta$.

(e) Write down the formula to find out the volume by cylindrical shells about the y-axis.

(f) The coordinates of foci of the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{16} = 1$ are ________ and ________.

(g) $[\vec{a},\ 2\vec{a},\ 3\vec{a}]$ = ________.

(h) $\lim_{t \to \infty} \left\langle \frac{t^3+1}{4t^3+2}, \frac{1}{t} \right\rangle$ = ________.

GROUP - B

2. Answer any eight of the following. [$1\frac{1}{2} \times 8$

(a) State Leibnitz's rule.

(b) Find out the parallel asymptote(s) of the curve $x^3 + y^3 - 3axy = 0$.

(c) How to know whether the curve $r = f(\theta)$ is symmetrical about the pole or not ?

(d) Evaluate $\int_0^{\pi/2} \sin^8 x \, dx$.

(e) Evaluate $\int \sec^3 x \, dx$.

(f) Evaluate $\int_0^1 x^2 (1 - x^2)^{3/2} \, dx$.

(g) Find the volume of the solid that is obtained when the region under the curve $y = \sqrt{x}$ over the interval [1, 4] is revolved about the x-axis.

(h) Find the new coordinates of the point (2, 4) if the coordinate axes are rotated through an angle $\theta = 30^0$.

(i) Let $\vec{r}(t) = t^2\hat{i} + e^t\hat{j} - (2\cos\pi t)\hat{k}$, then find $\int_0^1 \vec{r}(t)\,dt$.

(j) Prove that $\frac{d}{dt}[\vec{r}_1(t) + \vec{r}_2(t)] = \frac{d}{dt}[\vec{r}_1(t)] + \frac{d}{dt}[\vec{r}_2(t)]$.

GROUP - C

3. Answer any eight of the following. [2 × 8

(a) Find the points of inflexion on the curve $y = (\log x)^3$.

(b) Determine $\lim_{x \to 0}\left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right)$.

(c) Evaluate $\int \frac{dx}{x^{1/2} + x^{1/3}}$.

(d) Obtain a reduction formula for $\int x^m \sin\, nx\, dx$.

(e) Find the arc length of the curve $y = 3x^{3/2} - 1$ from x = 0 to x = 1.

(f) Find the area of the surface generated by revolving the curve $y = \sqrt{4 - x^2},\ -1 \le x \le 1$ aboout the x-axis.

(g) Find the rectangular coordinates of the point P whose polar cordinates are $(r, \theta) = \left(6, \frac{2\pi}{3}\right)$. Draw a rough graph of it.

(h) Calculate the scalar triple product $\vec{u}.(\vec{v} \times \vec{w})$ and the vector triple product $\vec{u} \times (\vec{v} \times \vec{w})$ of the vectors $\vec{u} = 3\hat{i} - 2\hat{j} - 5\hat{k}$, $\vec{v} = \hat{i} + 4\hat{j} - 4\hat{k}$, $\vec{w} = 3\hat{j} + 2\hat{k}$.

(i) Find $\vec{r}(t)$, given that $\vec{r}'(t) = \langle 3, 2t \rangle$ and $\vec{r}(1) = \langle 2, 5 \rangle$.

(j) If $\vec{r}(t)$ is a differentiable vector-valued function in 2-space or 3-space, and $\|\vec{r}(t)\|$ is constant for all t, then prove that $\vec{r}(t).\vec{r}'(t) = 0$.

GROUP - D

*Answer **any four** questions.*

4. Find $(x^2 e^x \cos x)_n$. [6

5. Trace : $y^2(a - x) = x^3$, $a > 0$. [6

6. If $I_n = \int_0^{\pi/3} \tan^n x \, dx$, then show that $(n - 1)(I_n + I_{n-2}) = (\sqrt{3})^{n-1}$. [6

7. Use cylindrical shells to find the volume of the solid generated when the region under $y = x^2$ over the interval [0, 2] is rotated about the line $y = -1$. [6

8. Find out the surface area of a sphere of radius r. [6

9. Let $\vec{u}$, $\vec{v}$, $\vec{w}$ are three non-zero vectors in 3-space. Prove that the volume V of the parallelopiped that has $\vec{u}$, $\vec{v}$ and $\vec{w}$ as adjacent edges is $V = |\vec{u} \cdot (\vec{v} \times \vec{w})|$. [6

10. Find out the tangential and normal components of acceleration. [6